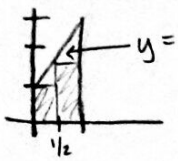
Quiz 21

November 18, 2016

Show all work and circle your final answer.

1. *(10 points)*

   (a) Find the left Riemann sum used to estimate the area under the graph of $f(x) = 3x + 1$ and above the x-axis between $x = 0$ and $x = 1$ using two rectangles.



$\Delta x = \frac{1-0}{2} = \frac{1}{2}$

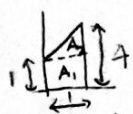
$R_2 = \frac{1}{2}\left[f(0) + f\left(\frac{1}{2}\right)\right]$

$= \frac{1}{2}\left[1 + \frac{5}{2}\right] = \boxed{\frac{7}{4}}$ (or 1.75)

   (b) Is your answer an over-estimate or an under-estimate? Why?

This is an underestimate because some of the area under the curve $y = 3x+1$ isn't covered by the two rectangles.

   (c) Evaluate $\int_0^1 3x + 1\,dx$.

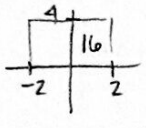


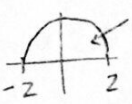
$A_{trap} = \frac{1}{2}(b_1+b_2)h = \frac{1}{2}(1+4)(1) = \boxed{5/2}$

OR $A_1 = 1(1)$, $A_2 = \frac{1}{2}(1)(3) = \frac{3}{2}$, so $A = \boxed{5/2}$

2. *(5 points)* Find $\int_{-2}^{2} 4 - \sqrt{4 - x^2}\,dx$.

$= \int_{-2}^{2} 4\,dx - \int_{-2}^{2} \sqrt{4-x^2}\,dx$

$=$ 

$-$ 

$A = \frac{1}{2}\pi(2)^2 = 2\pi$

$= \boxed{16 - 2\pi}$

3. *(5 points)* If $\int_{-1}^{4} f(x)\,dx = -3$ and $\int_{4}^{1} f(x)\,dx = 4$, evaluate $\int_{-1}^{1} f(x)\,dx$.

$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{4} f(x)\,dx - \int_{1}^{4} f(x)\,dx$

$= -3 - (-4)$

$= \boxed{1}$